PERGAMON

International Journal of Solids and Structures 37 (2000) 2887–2900

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

Strength prediction of composite laminates with multiple elliptical holes

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Received 22 November 1997; received in revised form 7 December 1998

Abstract

Based on the classical laminated plate theory, a finite composite plate with multiple elliptical holes is treated as an anisotropic multiple connected plate. Using the complex potential method in the plane theory of elasticity of an anisotropic body, a series solution to the title problem is obtained by means of the Faber series expansion, the conformal mapping and the least squares boundary collocation techniques. Laminate strength is predicted by using the concept of characteristic curve and the Yamada–Sun failure criterion. The effects of the layups, the hole sizes, the ellipticity of the holes, the loading conditions, the relative distance between holes, the total number of holes and their locations on the strength of laminates are studied in detail. Some useful conclusions are drawn. © 2000 Elsevier Science Ltd. All rights reserved.

1. Introduction

Composite materials, because of their high specific strength and stiffness and their flexible anisotropic property, have found a variety of applications in many engineering fields, such as in aerospace, automobiles and chemistry. As is known, holes and cutouts are inherent features and, in some cases, in the form of flaws in many engineering structures and these could cause serious problems of stress concentrations due to the geometry discontinuity. These problems are even more serious in structures made of composite materials, since the materials exhibit anisotropic behavior. Moreover, some composite materials are considered to be relatively brittle and, therefore, they are very sensitive to stress concentrations and could fail easily when the applied stress reaches the proportional limit. With this in view, a lot of attention has been paid by many researchers to predict the strength of composite structures where stress concentrations occur. The closed form solution to stress concentration around a circular hole in an infinite orthotropic plate was first obtained by using the complex potential method

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PII: S0020-7683(99)00033-5

(Lekhnitskii, 1957; Savin, 1961). Kosmodaminaskii and Chernic (1981) obtained the stress states of an infinite plate weakened by two elliptical holes with parallel axes. Extensive studies have also been made by the authors and their colleagues (Fan and Wu, 1988; Xu et al., 1992a; Xu, 1992b; Xu and Fan, 1991; Xu and Fan, 1993a, 1993b) on the thermoelasticity problems of infinite laminated plates with multiple elliptical holes. Gerhardt (1984) obtained the solution to a finite plate weakened by an circular hole using the hybrid finite element method. Garbo and Ogonowski (1980) and Garbo (1980) predicted the strength of infinite composite laminates with an unloaded fastener hole using a closed-form analytic approach. Ogonowski (1980) and Lin and Ko (1988) have studied similar problems of finite composite laminates by using the boundary collocation approach. These methods, however, still suffer some drawbacks, such as large data preparations, long CPU time and low accuracy. It is very difficult to analyze the stress and strength of a laminated plate with multiple holes, especially for finite plates because it involved a multiple connected boundary value problem. According to the authors' knowledge, the solution to the problem is still lacking. This is an important subject because open holes and mechanically multi-fastened joints are widely employed in composite structures. Recently, the authors (Xu et al., 1995a, 1995b, 1995c, 1999) have extensively studied the stress concentration of a finite laminated plate with multiple holes using conformal mapping, the Faber series expansion and the least squares boundary collocation technique. However, the results of the previous studies are still insufficient to be employed to design a composite structure and evaluate its strength because of the anisotropic behavior of composite materials. Thus, the objective of this paper is to present a practical approach for the title subject.

In this paper, based on the classical laminated plate theory (Jones, 1975), a finite composite plate with multiple elliptical holes is treated as an anisotropic multiple connected plate. The analytical study of the stress states around holes in a finite composite laminated plate will be based on the complex potential method in the plane theory of elasticity of an anisotropic body with the aid of the Faber series expansion, the conformal mapping and the least squares boundary collocation techniques (Xu et al., 1995a, 1995b, 1995c, 1999). Laminate strength is predicted by using the concept of characteristic curve (Whitney and Nuismer, 1974) and the Yamada–Sun failure criterion (Yamada and Sun, 1978). Furthermore, the effects of the layups, the hole sizes, the ellipticity of the holes, the loading conditions, the relative distance between holes, the total number of holes and their locations on the strength of laminates are studied in detail. It is worth pointing out that the present method can also be adopted in some advanced materials, such as ceramic composites, metal matrix composites and isotropic materials. The characteristic curve may be used to evaluate the effect of the quality of the machined holes on the mechanical properties of the composite material.

2. Method of stress analysis

In the classical theory, the laminated composite plate is treated as an anisotropic plate. Therefore, the constitutive equation for a laminated plate in plane stress is (Jones, 1975; Xu, 1992b)

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} \begin{Bmatrix} \bar{\sigma}_x \\ \bar{\sigma}_y \\ \bar{\sigma}_{xy} \end{Bmatrix}, \tag{1}$$

where $\{\bar{\sigma}\}$ is the average in-plane stress and a_{ij} are the equivalent compliance coefficients depending on the fibre orientation, the stacking sequence and the property of each lamina. Therefore, the plane problem of laminated composite materials can be solved by using the complex potential method in the plane theory of elasticity for an anisotropic body.

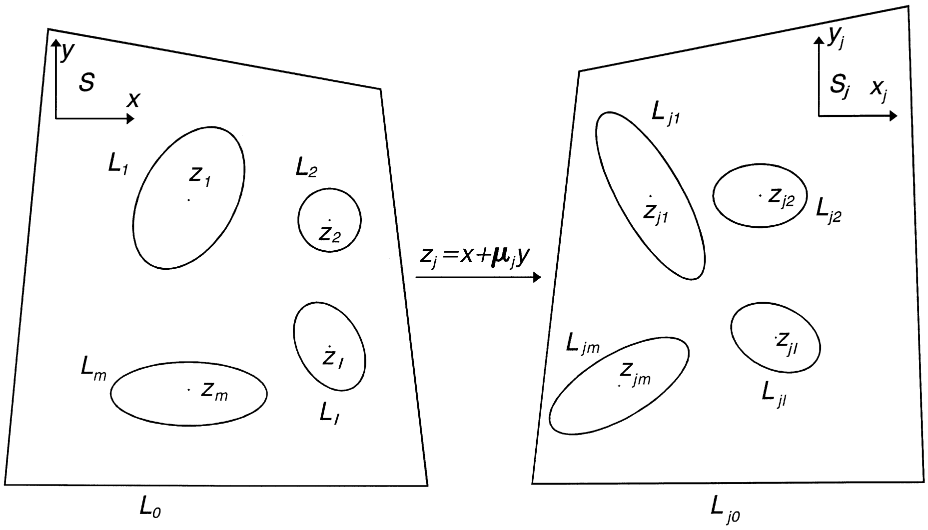

Fig. 1. A finite composite laminated plate weakened by multiple elliptical holes.

Consider a finite composite laminated plate weakened by multiple elliptical holes with contours L_0, L_1, L_2,..., L_1, as shown in Fig. 1. The semi-major, semi-minor axes and centers of holes are denoted by a_m, b_m and z_m (m = 1,2,..., l), respectively. The affine transformation, $z_j = x + \mu_j y$, transforms region S onto region S_j, the point z_m in the region S corresponds to point z_{jm} in the region S_j. Let the principal vector of forces acting on the contour of every hole be equal to zero, the complex potential function, $\varphi_j(z_j)$, can be expressed as (Xu et al., 1995a, 1995b, 1995c, 1999):

$$\varphi_j(z_j) = \varphi_{0j}(z_j) + \sum_{k=0}^{\infty} b_{jk} P_k(z_j)\ (j = 1,2), \tag{2}$$

where $\varphi_{0j}(z_j)$ is a holomorphic function in the infinite region with l number of elliptical holes. $P_k(z_j)$is the Faber polynomial of the region limited by contour L_{j0}.

The mapping function is given as follows:

$$z_j - z_{jm} = R_{jm}\left(\xi_{jm} + \frac{t_{jm}}{\xi_{jm}}\right)\ (m = 1,2,\ldots, j = 1,2), \tag{3}$$

where

$$R_{jm} = \frac{a_m i \mu_j b_m}{2}$$

and

$$t_{jm} = \frac{a_m + i\mu_j b_m}{a_m i \mu_j b_m}.$$

This mapping function transforms the exterior of hole m in the complex plane z_j into the exterior of a unit circle, $\xi_{jm} = \exp(i\theta)$, in the complex plane ξ_{jm}. If the major and minor axes are not parallel to the coordinate axes, the rotational mapping should be used.

Using Laurent series expansion and the Faber polynomial in the general region, the complex potential function can be shown as (Xu et al., 1995a, 1995b, 1995c, 1999):

$$\varphi_j(z_j)=\sum_{m=l}^{l}\sum_{k=l}^{\infty}\frac{b_{jmk}}{\xi_{jm}^{k}}+\sum_{k=0}^{\infty}a_{jk}z_j^k. \tag{4}$$

Obviously, the complex potential function Eq. (4) is analytic in the region S_j. Once the unknown coefficients, b_{jmk} and a_{jk}, are determined by using the boundary conditions, the stress and the displacement field can be obtained uniquely.

Assuming that the external forces, X_n and Y_n, are applied or the displacements, $u(t)$ and $v(t)$, are given on the contour, the boundary conditions can be expressed as (Xu et al., 1995a, 1995b, 1995c, 1999):

$$\sum_{j=l}^{2}\left[r_j\varphi_j(z_j)+s_j\overline{\varphi_j(z_j)}\right]=f(t), \tag{5}$$

where

$$r_j=1+i\mu_j,$$

$$s_j=1+i\overline{\mu_j}$$

and

$$f(t)=\pm\int_0^t i(X_n+iY_n)\mathrm{d}s+c_i+ic_2$$

when the surface forces are given. The upper and lower signs correspond to the outer and inner contours and

$$r_j=p_j+iq_j,$$

$$s_j=\overline{p_j}+i\,\overline{q_j}$$

and

$$f(t)=u(t)+iv(t)+i(v_0+\omega x)+u_0-\omega y$$

if the displacements are prescribed on the boundary.

The right-hand-side part of Eq. (5) can be expanded into the complex Fourier series, which is a power series of variable $\sigma=\exp(i\theta)$.

From the mapping function of Eq. (3), it can be seen that function $\xi_{jm}(z_j)$ is holomorphic in the complex plane z_j weakened by the m-th hole. Therefore, function $\xi_{jm}^n(z_j)$ is holomorphic in the interior of the p-th ($p\neq m$) hole and continuous to its boundary. Thus they can be expanded into a Faber series:

$$\xi_{jm}^n=\sum_{k=0}^{\infty}A_{n,k}^{jm}P_{kp}(z_j). \tag{6}$$

Similarly,

$$z_j^n = \sum_{k=0}^{\infty} H_{n,k}^{\,j} P_{kp}(z_j), \tag{7}$$

where $P_{kp}(z_j)$ is the k-th Faber polynomial for the ellipse L_{jp} of the complex z_j plane and

$$P_{kp}(z_j) = \xi_{jp}^k + \frac{t_{jp}^k}{\xi_{jp}^k},$$

$$P_{0p} = 1, \tag{8}$$

where the coefficients $A_{n,k}^{jm}$, $H_{n,k}^{\,j}$ in the Faber series can be determined by the Fourier expansion method (Xu, 1992b). Substituting Eqs. (6) and (7) into the complex potential expression (4) and using $\xi_{jp} = \exp(i\theta) = \sigma$ in the contour L_{jp} of the p-th hole, the boundary values of $\varphi_j(z_j)$ ($j = 1,2$) are obtained in a power series of σ.

It is easy to prove (Xu, 1992b) that the point $z = z_m + a_m \cos\theta + ib_m \sin\theta$ on the physical region can be transformed into the point $\sigma = \exp(i\theta)$ on the ξ_{jm} plane by using the affine transformation $z_j = x + \mu_j y$ and the mapping transformation, Eq. (3). Taking the values of $\varphi_j(z_j)$ and performing a partial sum up to the N-th power term, then, substitute them into the boundary condition of every elliptical hole and equate the coefficients of the same power σ^k ($k = 0, \pm 1, \pm 2, \ldots, \pm N$) on both sides of every equation, we obtain the coefficients, b_{jmk}, a_{jk} and C_p ($p = 1,2,\ldots, l$), for the $(2N + 1)l$ linear equations. But these linear equations obtained from inner boundary conditions are not enough to determine all the coefficients. Therefore, it is necessary to use the outer boundary conditions. Although, for the smooth outer contour L_0 of the plate, an accurate solution can be obtained by using the Faber polynomial in the general region, the calculation is lengthy, complicated and troublesome. A more convenient method, based on the least square boundary collocation technique, is used in this paper. Taking the collocation points z_{ck} ($k = 1,2,\ldots, M, M \geq 2N + 4$) along the outer contour L_0 and substituting z_{ck} into the boundary condition of Eq. (5), we can obtain the unknown coefficients, b_{mjk} and a_{jk}, that satisfy the outer boundary conditions for the linear equations. These equations together with $(2N + 1)l$ equations that satisfy the inner boundary conditions are used to determine the complex potential function $\varphi_j(z_j)$. The stress field in the laminated plate can then be computed by the following equations, and its strain field will be determined by Eq. (1).

$$\sigma_x = 2\,\mathrm{Re} \sum_{j=1}^{2} \mu_j^2 \varphi_j'(z_j),$$

$$\sigma_y = 2\,\mathrm{Re} \sum_{j=1}^{2} \varphi_j'(z_j)$$

and

$$\tau_{xy} = -2\,\mathrm{Re} \sum_{j=1}^{2} \mu_j \varphi(z_j). \tag{9}$$

Obviously, the complex potential function $\varphi_j(z_j)$ is an analytic function in the region S_j. Therefore, the accuracy of the solution can be ascertained according to whether the boundary conditions are satisfied fully or not. In the present method, the inner boundary conditions can be satisfied with an absolute error of less than 10^{-5}. By increasing the number of collocation points, the outer boundary conditions can be better satisfied and a relative error less than one percent can be ensured. As we know from the Saint–Venant principle, very accurate results (Xu et al., 1995a, 1995b, 1995c, 1999) of stress distribution around holes can be obtained by using the present method, that is the main concern of many researchers.

3. Prediction of failure

When the stress state of laminated plates is determined, the failure strength and the failure initiation location can be predicted by employing a failure criterion. To account for the inelastic or nonlinear behavior at the hole boundary of the composite laminate, the characteristic dimension concept proposed by Whitney and Nuismer (1974) has been incorporated with Yamada–Sun (Yamada and Sun, 1978) failure criterion in this paper. Laminate failure is predicted by comparing the elastic stress distributions with the Yamada–Sun failure criterion,

$$\left(\frac{\sigma_1}{X}\right)^2+\left(\frac{\tau_{12}}{S}\right)^2=e^2,\ \left\{\begin{array}{ll} e>1 & \text{failure} \\ e=1 & \text{critical} \\ e<1 & \text{safe} \end{array}\right\}, \tag{10}$$

on a ply-by-ply basis at a characteristic dimension away from every hole boundary ($r=R_c$). The parameter X in Eq. (10) is the axial strength of a single layer, S is the rail-shear strength of a symmetric cross-ply laminate and σ_1, τ_{12} are the laminas stresses in the principle material axes which can be easily determined by the laminated plate theory (Jones, 1975) when the strain of the laminate is given. Laminate failure is assumed to occur when the first ply fails, as is predicted by Eq. (10).

In this paper, the predictions on strength were made by the following procedures:

1. calculate the laminate stiffness using classical laminated plate theory,
2. calculate the laminate stresses and strains in relation to the load factor using the method described in the previous section,
3. determine the relative laminate stresses in the principal material axes by means of the classical laminated plate theory,
4. use the Yamada–Sun criterion (Yamada and Sun, 1978) on a ply-by-ply basis and apply the characteristic curve of every hole to get the value e which is proportional to the applied load p,
5. calculate the failure load by the formula:

$$p_f=\frac{p}{e},$$

where p_f is the predicted laminate strength.

4. Numerical results

The numerical examples presented in this paper have focused on the strength predictions of composite laminated plates with multiple elliptical holes. The stress concentration of laminates weakened by multiple holes has been discussed elsewhere by the authors (Xu et al., 1995a, 1995b, 1999). The partial

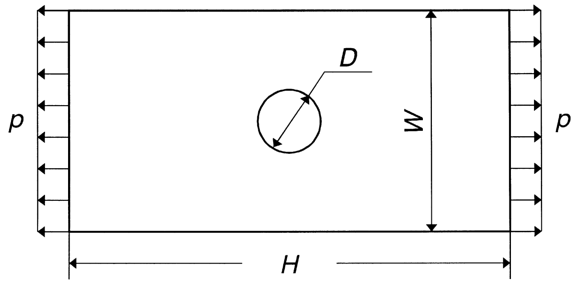

Fig. 2. A finite laminated plate with a hole of diameter D.

sum N and the number of collocation points on the outer boundary are taken to be 10 and 32, respectively. Numerical results indicate that the boundary conditions are satisfied approximately but the error is less than one percent.

Consider a finite laminated plate with a hole, where $D = 0.25''$, $H = 11''$, $W/D = 6.0$, as shown in Fig. 2. The laminate is composed of graphite/epoxy. The properties of the lamina material (Garbo and Ogonowski, 1980; Garbo, 1980) are given below

$E_1 = 18.69$ Msi, $E_2 = 1.90$ Msi, $G_{12} = 0.85$ Msi, $\nu_{12} = 0.3$
$X_t = 232.5$ Ksi, $X_c = 291.7$ Ksi, $S = 17.3$ Ksi
$R_c = 0.02''$ (tension), $R_c = 0.0225''$ (shear), $R_c = 0.025''$ compression

A comparison of the present results for strength predictions of the laminated plates with various layups ($0_\alpha / \pm 45_\beta / 90_\gamma$) containing a central hole and the experimental data obtained by Garbo and Ogonowski (1980) and Garbo (1980) is given in Table 1. The results show that a reasonably good agreement between the numerical predications and the experimental data is obtained. Especially for the tensile strength, the maximum relative error is only 3.7 percent. The results also prove that it is reasonable to treat the characteristic dimension as a constant of the material in engineering analysis. The results indicate that the strength strongly depends on the percentage of different layups of the whole plate. As it is expected, the strength along the x direction increases rapidly with an increase of the number of 0° lamina and an increase of number of ±45° lamina will increase the strength for the

Table 1
A comparison of the present strength predictions and the experimental results (Units: Ksi)

Layups ($0_\alpha / \pm 45_\beta / 90_\gamma$)	Tension strength		Compression strength	
	Predictions	Experiments (Garbo and Ogonowski, 1980)	Predictions	Experiments (Garbo, 1980)
70/20/10	76.62	76.36	94.29	98.10
50/40/10	63.85	65.73	85.20	75.24
30/60/10	47.58	48.65	61.06	60.00
50/10/40	53.43	55.14	66.71	77.14
40/20/40	52.87	54.05	68.83	65.76
40/50/10	55.72	56.40	73.84	66.70
30/30/40	45.52	46.49	61.05	54.29
20/40/40	38.14	36.76	50.79	45.71
20/70/10	39.32	40.00	48.65	45.60
10/80/10	31.00	31.20	36.63	38.10

laminates when the number of 0° layup is fixed. These are due to the anisotropic strength behavior of the laminate and an increase in the number of ±45° lamina reduces the stress concentration.

Fig. 3(a,b) shows the effect of the diameter D of the hole on the tensile and compressive strengths of the laminate with various layups, as shown in Fig. 2. The strength of the laminates decreases rapidly with an increase in the diameter D of the hole and the decrease becomes small when $D \geq 0.25''$. The reason is that the increase of the diameter of the hole shortens the relative distance R_c/D between the characteristic curve and the boundary of the hole, and so the stress at the characteristic curve increases.

The strength of the square laminate (50/40/10) (Fig. 4) with a central hole under tension, compression and shear loading is also studied. Suppose the laminate contains a central, circular hole of diameter $D = 0.25''$. The effect of the relative plate size W/D on the strength of the laminate is illustrated in Fig. 5. The strength rises rapidly with an increase of W/D. Similar to the stress concentration effect (Xu et al., 1995b), the increase will be small when $W/D \geq 5.0$. For the laminated plate subjected to the three different loading conditions, the compressive strength is the highest and the shear strength is the lowest. This is partly due to the fact that the stress concentration is most severe under shear loading. Fig. 6 demonstrates the effect of ellipticity on the strength of the laminate with a center elliptical hole. In general, the strength decreases with an increase in ellipticity, especially when $a/b < 1.0$. It is interesting to note that the tensile, the compressive and the shear strengths reached a maximum when a/b is at about 0.5, 0.75 and 0.75, respectively. The value of a/b strongly depends on the layups of the plate and its loading conditions. For a specific laminated plate subjected to a specific load, there exists a hole of specific ellipticity with which the strength of the laminate is the highest, in general it is not a circular hole. It is worth saying that when $a/b = 0$, the problem is mathematically reduced to a crack in the finite plate. Thus, it is very convenient to calculate the strength of laminates with multiple notches and cracks.

The following discussion focuses on the strength of laminate (50/40/10) with multiple holes (as shown in Figs. 7–9) under the loading of tension, compression and shear in the x direction coinciding with the one illustrated in Fig. 1. Suppose $D = 0.25''$, $E/D = T/D = 2.5$ and $L/D = 1.5$. This condition applies except when the effect of the relative center-to-center distance between holes is studied. Fig. 10 illustrates the effects of the relative distance L/D on the strength of the laminate with two holes in series [Fig. 7(a)]. When the relative center-to-center distance becomes smaller, the shear strength decreases rapidly and, conversely, the tensile and the compressive strength increase rapidly. When $L/D \geq 4.5$, the effect of the relative center-to-center distance on the strength becomes very small. In this case, the plate with multiple holes can be treated as with a single hole in engineering analysis. Fig. 11 shows the strength of the plate with multiple holes in series [Figs. 7(a), 8(a), and 9(a)]. The results indicate that an increase in the number of holes has caused the tensile and compressive strengths of the laminate to be increased, but a significant reduction in shear strength results.

These conclusions are consistent with that for the stress concentration obtained previously (Xu et al., 1995a). Table 2 describes the effect of hole position (Figs. 7–9) on the strength of the laminate. The

Table 2
The strength of laminate (50/40/10) with multiple holes (Units: Ksi)

Hole position	Two holes		Three holes			Four holes	
	a	*b*	*a*	*b*	*c*	*a*	*b*
Tension	65.19	47.44	67.23	49.08	40.89	67.77	51.27
Compression	83.79	61.48	86.01	62.92	54.77	86.55	66.07
Shear	23.65	21.82	20.10	20.27	19.21	17.50	19.47

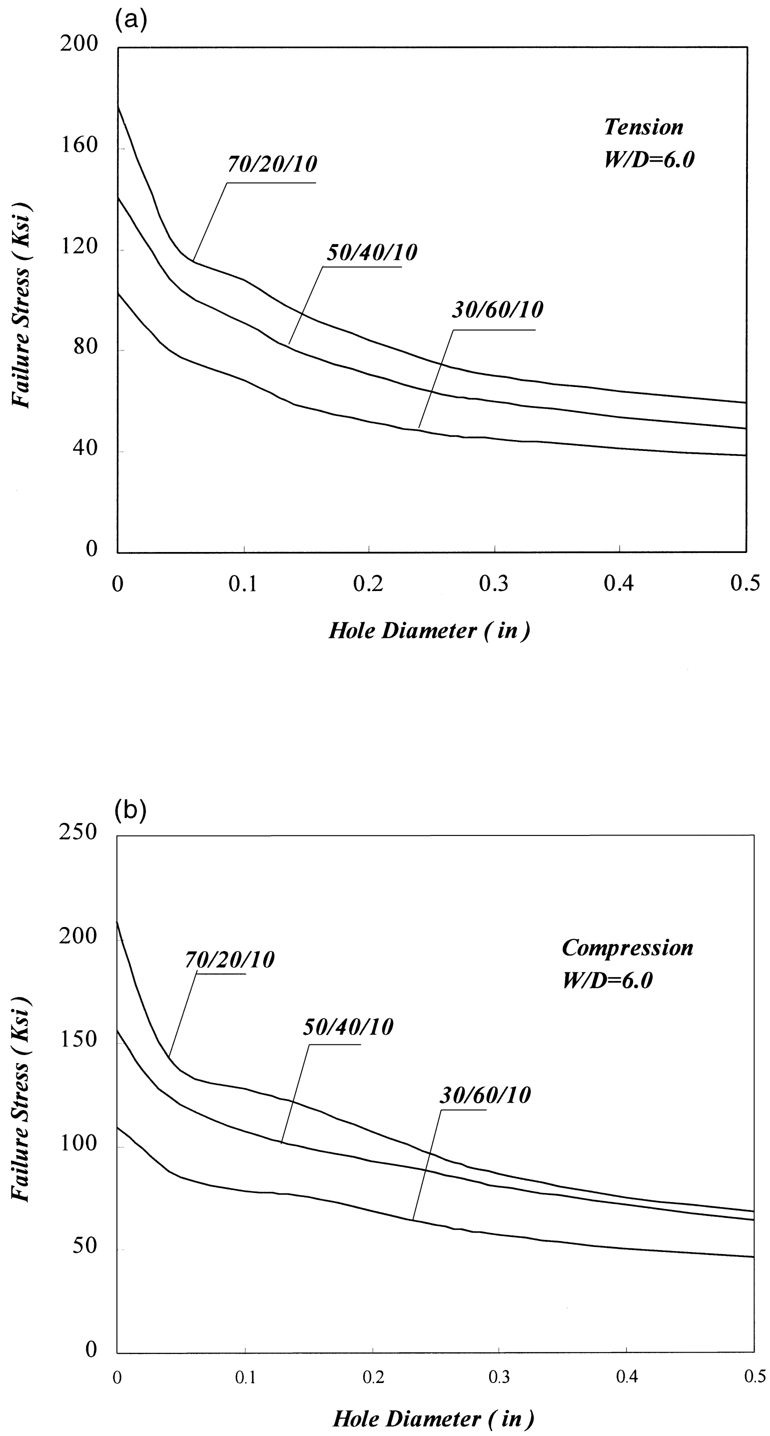

Fig. 3. The effect of hole diameter D on strength.

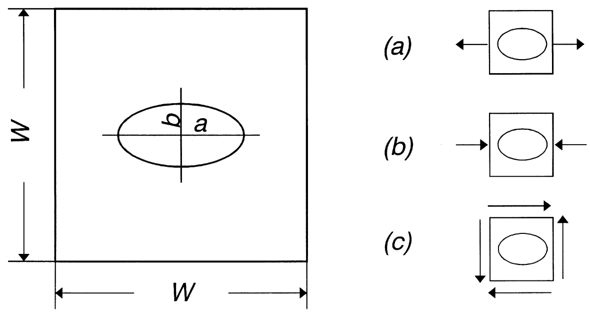

Fig. 4. A square laminate with a centric elliptical hole.

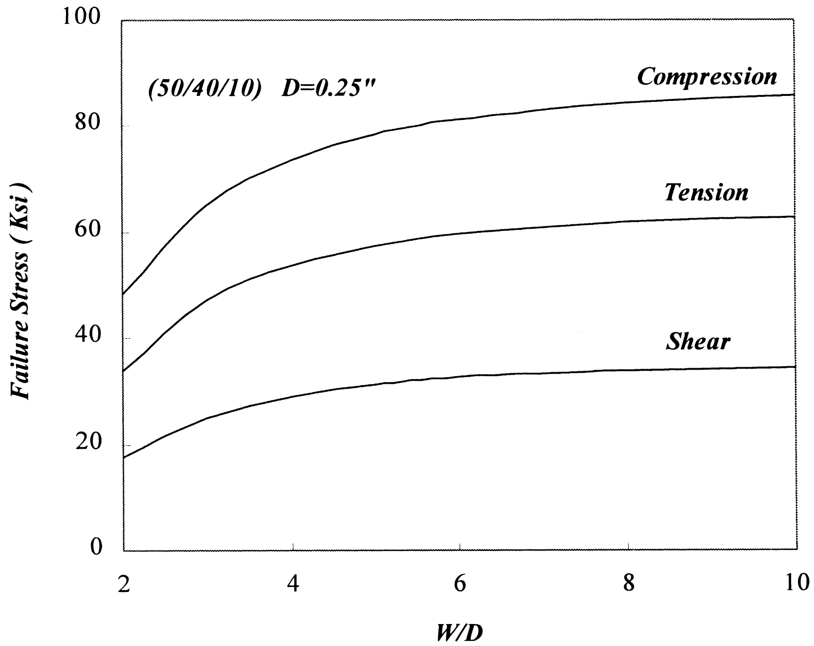

Fig. 5. The effect of the relative plate size W/D on the strength.

location of the hole has been found to have a significant effect on the tensile and compressive strength, and the effect on the shear strength is small. For the tensile and compressive strengths, the highest value is obtained when the holes are in series.

5. Conclusions

Based on the results obtained in this study, the following conclusions may be drawn:

1. The strength of a laminate with multiple holes strongly depends on its layups, hole size and loading condition. Increase the layup along the direction of the applied force would increase the strength

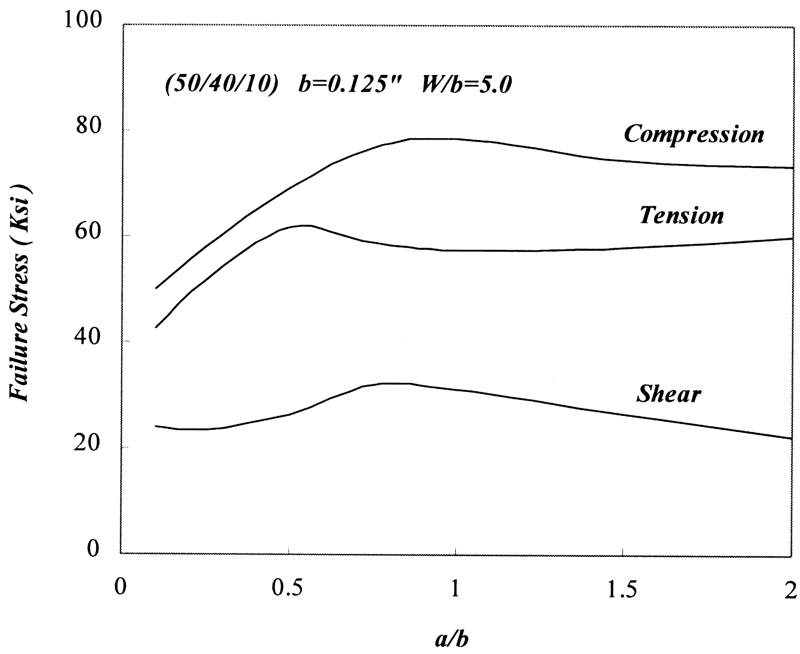

Fig. 6. The effect of the ellipiticity a/b on the strength.

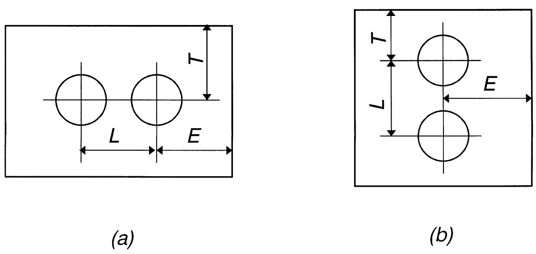

Fig. 7. A finite laminate weakened by two circular holes.

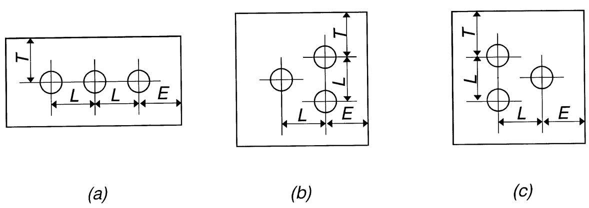

Fig. 8. A finite laminate weakened by three circular holes.

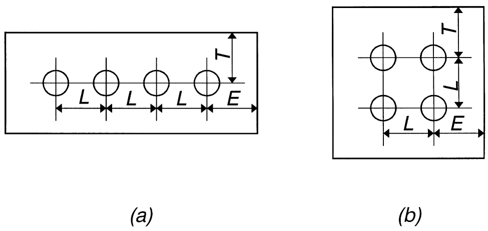

Fig. 9. A finite laminate weakened by four circular holes.

rapidly. In general, increase the numbers of the $\pm 45^\circ$ lamina would cause a raise in strength. The laminate strength is weakened by the presence of holes and the larger the diameter of the hole, the smaller the strength will be.

2. The relative size of the plate W/D has a strong effect on the strength, the strength increases rapidly with an increase of the relative size. When $W/D \geq 5.0$, it is reasonable to treat the finite plate as an infinite plate in most engineering analyses.
3. With the increase of ellipticity, the strength becomes smaller. For a specific laminated plate subjected to a specific load, there exists a hole of specific ellipticity with which the strength of the laminate is the highest, in general it is not a circular hole.
4. The effect of the relative center-to-center distance, L/D, on strength is obvious. In general, the shorter the relative distance, the lower the strength. But if the direction of the series of holes is same as that of the applied force, a opposite effect is obtained.

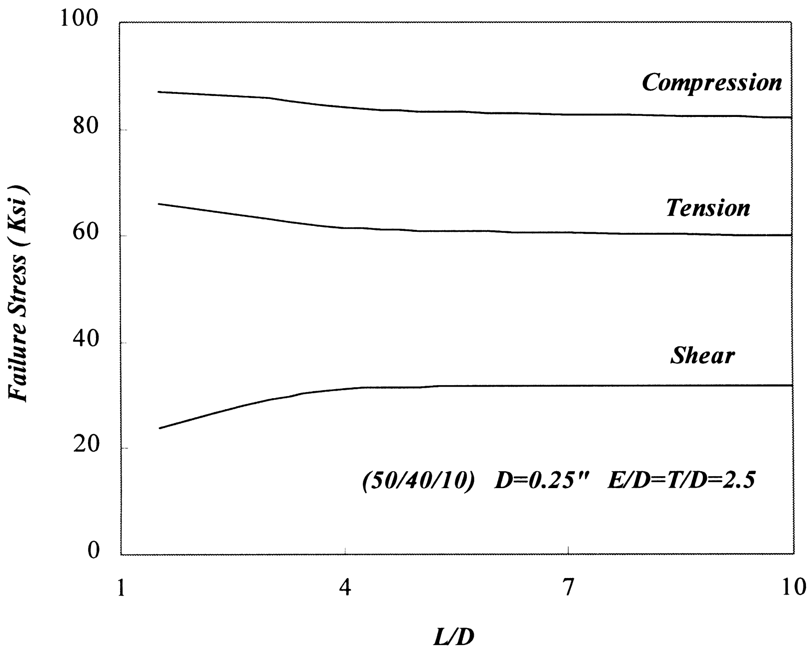

Fig. 10. The effect of the relative center-to-center distance L/D on the strength.

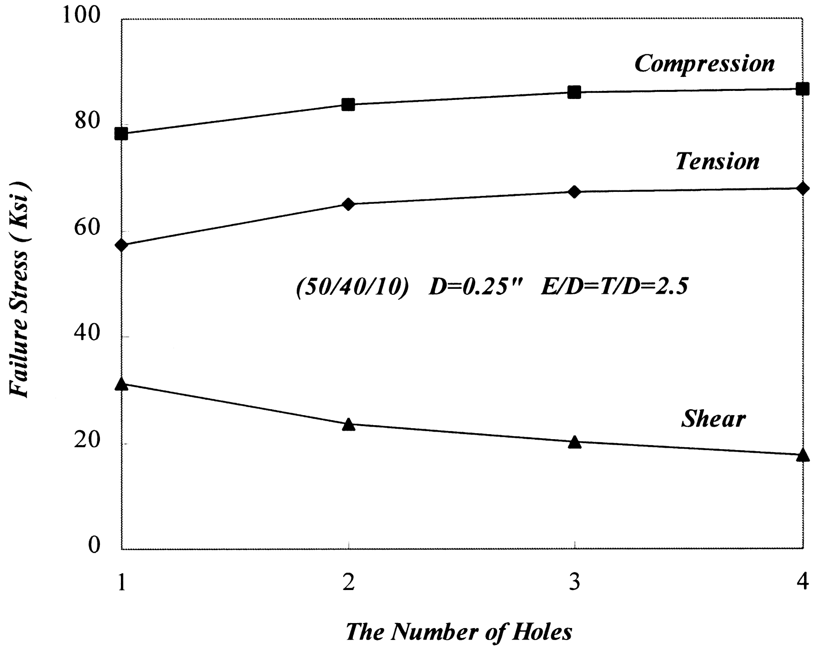

Fig. 11. The effect of the number of holes on the strength.

5. An increase in the number of holes in the direction of the applied force increases the strength, but in other cases, it could reduce the strength.
6. The position of hole has a significant effect on the strength. For the tensile and compressive strength, the highest strength is obtained when the holes is in series, however the effect is small on shear strength.
7. The present method is very efficient for predicting the strength of a finite composite laminate weakened by multiple elliptical holes, notches or cracks. It has many advantages such as high accuracy, short computer time and convenience of use. In general, it is reasonable to consider the characteristic dimension as a constant of the material in most engineering analyses.

Acknowledgements

This work described in this paper was fully supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China (Project No. HKP 107/95E).

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